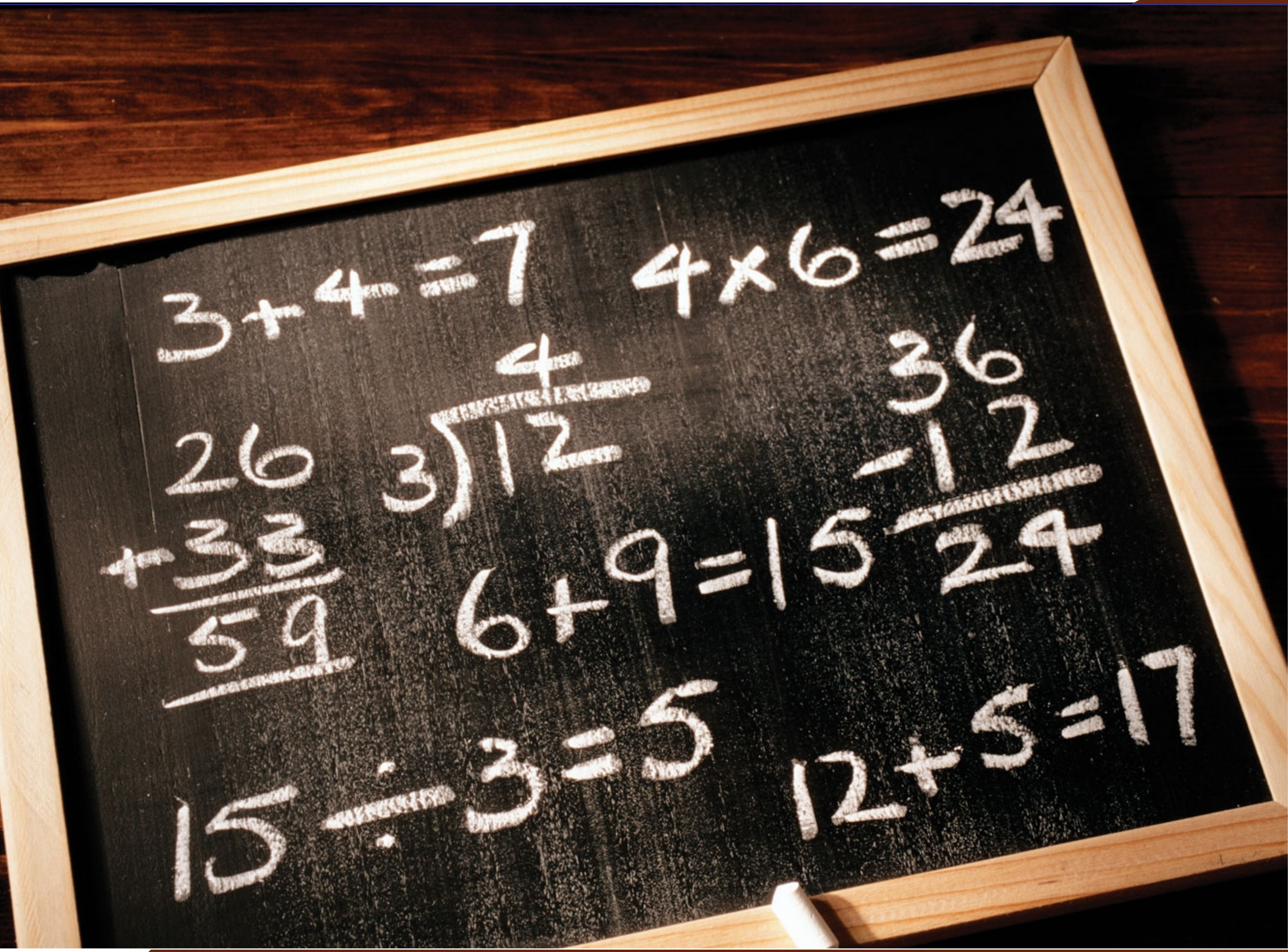
LA LAQSHYA ©

A C A D E M Y

ISO 9001:2008 INSTITUTE



New

Quantitative Aptitude

Edition 2017

Study Material
For

Quantitative Aptitude

Regd. Office :- A-202, Shanti Enclave, Opp.Railway Station, Mira Road(E), Mumbai.
www.bankpo.laqshya.in | bankpo@laqshya.in

Quantitative Aptitude For IBPS PO / SBI PO / Bank Clerical Exams

INDEX

1. NUMBER SYSTEM

A. TYPES OF NUMBERS

1. **Natural Numbers :**Counting numbers 1, 2, 3, 4, 5,..... are called natural numbers.

2. **Whole Numbers :**All counting numbers together with zero form the set of whole numbers. Thus,
(i) 0 is the only whole number which is not a natural number.
(ii) Every natural number is a whole number.

3. **Integers :** All natural numbers, 0 and negatives of counting numbers i.e., {…, - 3 , - 2 , - 1 , 0, 1, 2, 3,…..} together form the set of integers.

(i) Positive Integers : {1, 2, 3, 4, …..} is the set of all positive integers.
(ii) Negative Integers : {- 1, - 2, - 3,…..} is the set of all negative integers. (iii) Non-Positive and Non-Negative Integers : 0 is neither positive nor negative. So, {0, 1, 2, 3,….} represents the set of non-negative integers, while {0, - 1 , - 2 , - 3 , …..} represents the set of non-positive integers.

4. Even Numbers :A number divisible by 2 is called an even number, e.g., 2, 4, 6, 8, 10, etc.

5. Odd Numbers :A number not divisible by 2 is called an odd number. e.g., 1, 3, 5, 7, 9, 11, etc.

6. **Prime Numbers :**A number greater than 1 is called a prime number, if it has exactly two factors, namely 1 and the number itself. Prime numbers upto 100 are : 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97.

Prime numbers Greater than 100 : Let be a given number greater than 100. To find out whether it is prime or not, we use the following method :
Find a whole number nearly greater than the square root of p. Let k >square root of p. Test whether p is divisible by any prime number less than k. If yes, then p is not prime. Otherwise, p is prime.

e.g,,We have to find whether 191 is a prime number or not. Now, 14 >square root of 191.Prime numbers less than 14 are 2, 3, 5, 7, 11, 13.
191 is not divisible by any of them. So, 191 is a prime number.

7. **Composite Numbers :**Numbers greater than 1 which are not prime, are known as composite numbers, e.g., 4, 6, 8, 9, 10, 12.
Note :
(i) 1 is neither prime nor composite.
(ii) 2 is the only even number which is prime.
(iii) There are 25 prime numbers between 1 and 100.

Co-primes : Two numbers a and b are said to be co-primes, if their H.C.F. is 1. e.g., (2, 3), (4, 5), (7, 9), (8, 11), etc. are co-primes,

B. MULTIPLICATION BY SHORT CUT METHODS

1. Multiplication By Distributive Law :
(i) a* (b + c) = a * b + a * c (ii) a * (b-c) = a * b-a * c.

Ex.(i) 567958 x 99999 = 567958 x (100000 - 1) = 567958 x 100000 - 567958 x 1 = (56795800000 - 567958) = 56795232042.

(ii) 978 x 184 + 978 x 816 = 978 x (184 + 816) = 978 x 1000 = 978000.

2. **Multiplication of a Number By 5^n :** Put n zeros to the right of the multiplicand and divide the number so formed by 2^n
Ex. 975436 x 625 = 975436 x 5^4= 9754360000 =609647600

C. BASIC FORMULAE

(i) $(a + b)^2 = a^2 + b^2 + 2ab$
(ii) $(a - b)^2 = a^2 + b^2 - 2ab$
(iii) $(a + b)^2 - (a - b)^2 = 4ab$
(iv) $(a + b)^2 + (a - b)^2 = 2 (a^2 + b^2)$
(v) $(a^2 - b^2) = (a + b) (a - b)$
(vi) $(a + b + c)^2 = a^2 + b^2 + c^2 + 2 (ab + bc + ca)$
(vii) $(a^3 + b^3) = (a +b) (a^2 - ab + b^2)$
(viii) $(a^3 - b^3) = (a - b) (a^2 + ab + b^2)$
(ix) $(a^3 + b^3 + c^3 -3abc) = (a + b + c) (a^2 + b^2 + c^2 - ab - bc - ca)$
(x) If $a + b + c = 0$, then $a^3 + b^3 + c^3 = 3abc$.

D. DIVISION ALGORITHM OR EUCLIDEAN ALGORITHM
If we divide a given number by another number, then :
Dividend = (Divisor x Quotient) + Remainder

(i) $(x^n - a^n)$ is divisible by $(x - a)$ for all values of n.
(ii) $(x^n - a^n)$ is divisible by $(x + a)$ for all even values of n.
(iii) $(x^n + a^n)$ is divisible by $(x + a)$ for all odd values of n.

E. PROGRESSION - A succession of numbers formed and arranged in a definite order according to certain definite rule, is called a progression.

1. **Arithmetic Progression (A.P.) :**If each term of a progression differs from its preceding term by a constant, then such a progression is called an arithmetical progression. This constant difference is called the common difference of the A.P.

An A.P. with first term a and common difference d is given by a, (a + d), (a + 2d),(a + 3d),... The nth term of this A.P. is given by T_n =a (n - 1) d.

The sum of n terms of this A.P.

S_n = n/2 [2a + (n - 1) d] = n/2 (first term + last term).

SOME IMPORTANT RESULTS :

(i) $(1 + 2 + 3 + \ldots + n) = n(n+1)/2$
(ii) $(l^2 + 2^2 + 3^2 + \ldots + n^2) = n (n+1)(2n+1)/6$
(iii) $(1^3 + 2^3 + 3^3 + \ldots + n^3) = n^2(n+1)^2$

2. **Geometrical Progression (G.P.) :**A progression of numbers in which every term bears a constant ratio with its preceding term, is called a geometrical progression.

The constant ratio is called the common ratio of the G.P. A G.P. with first term a and common ratio r is : a, ar, ar^2,
In this G.P. $T_n = ar^{n-1}$
Sum of the n terms, $S_n = a(1-r^n) / (1-r)$

2. H.C.F. AND L.C.M.

A. **Factors and Multiples :**If a number a divides another number b exactly, we say that a is a factor of b. In this case, b is called a multiple of a.

B. **Highest Common Factor (H.C.F.) or Greatest Common Measure (G.C.M.) or Greatest Common Divisor (G.C.D.):** The H.C.F. of two or more than two numbers is the greatest number that divides each of them exactly.

There are two methods of finding the H.C.F. of a given set of numbers :

1. **Factorization Method :**Express each one of the given numbers as the product of prime factors.The product of least powers of common prime factors gives H.C.F.
2. **Division Method:** Suppose we have to find the H.C.F. of two given numbers. Divide the larger number by the smaller one. Now, divide the divisor by the remainder. Repeat the process of dividing the preceding number by the remainder last obtained till zero is obtained as remainder. The last divisor is the required H.C.F.
   Finding the H.C.F. of more than two numbers : Suppose we have to find the H.C.F. of three numbers. Then, H.C.F. of [(H.C.F. of any two) and (the third number)] gives the H.C.F. of three given numbers. Similarly, the H.C.F. of more than three numbers may be obtained.

C. **Least Common Multiple (L.C.M.) :**The least number which is exactly divisible by each one of the given numbers is called their L.C.M.

1. **Factorization Method of Finding L.C.M.:** Resolve each one of the given numbers into a product of prime factors. Then, L.C.M. is the product of highest powers of all the factors.
2. **Common Division Method {Short-cut Method} of Finding L.C.M.:** Arrange the given numbers in a row in any order. Divide by a number which divides exactly at least two of the given numbers and carry forward the numbers which are not divisible. Repeat the above process till no two of the numbers are divisible by the same number except 1. The product of the divisors and the undivided numbers is the required L.C.M. of the given numbers,

D. **Product of two numbers =Product of their H.C.F. and L.C.M.**

E. **Co-primes:** Two numbers are said to be co-primes if their H.C.F. is 1.

F. **H.C.F. and L.C.M. of Fractions:**

1. H C F= H.C.F. of Numerators / L.C.M. of Denominators
2. L C M = L.C.M of Numerators / H.C.F. of Denominators

G. **H.C.F. and L.C.M. of Decimal Fractions:** In given numbers, make the same number of decimal places by annexing zeros in some numbers, if necessary. Considering these numbers without decimal point, find H.C.F. or L.C.M. as the case may be. Now, in the result, mark off as many decimal places as are there in each of the given numbers.

Comparison of Fractions: Find the L.C.M. of the denominators of the given fractions. Convert each of the fractions into an equivalent fraction with L.C.M. as the denominator, by multiplying both the numerator and denominator by the same number. The resultant fraction with the greatest numerator is the greatest.

3. DECIMAL FRACTIONS

A. Decimal Fractions : Fractions in which denominators are powers of 10 are known as decimal fractions.
Thus ,1/10=1 tenth=0.1; 1/100=1 hundredth =0.01;
99/100=99 hundredths=0.99; 7/1000=7 thousandths=0.007,etc

B. Conversion of a Decimal Into Vulgar Fraction : Put 1 in the denominator under the decimal point and annex with it as many zeros as is the number of digits after the decimal point. Now, remove the decimal point and reduce the fraction to its lowest terms.
Thus, 0.25=25/100=1/4;2.008=2008/1000=251/125.

C. Annexing zeros to the extreme right of a decimal fraction does not change its value
Thus, 0.8 = 0.80 = 0.800, etc.

D. If numerator and denominator of a fraction contain the same number of decimal places, then we remove the decimal sign.
Thus, 1.84/2.99 = 184/299 = 8/13; 0.365/0.584 = 365/584=5

E. Operations on Decimal Fractions :

1. **Addition and Subtraction of Decimal Fractions :**The given numbers are so placed under each other that the decimal points lie in one column. The numbers so arranged can now be added or subtracted in the usual way.

2. **Multiplication of a Decimal Fraction By a Power of 10 :**Shift the decimal point to the right by as many places as is the power of 10.

   Thus, 5.9632 x 100 = 596,32; 0.073 x 10000 = 0.0730 x 10000 = 730.

3. **Multiplication of Decimal Fractions :**Multiply the given numbers considering them without the decimal point. Now, in the product, the decimal point is marked off to obtain as many places of decimal as is the sum of the number of decimal places in the given numbers.
   Suppose we have to find the product (.2 x .02 x .002). Now, 2x2x2 = 8.
   Sum of decimal places = (1 + 2 + 3) = .2 x .02 x .002 = .000008.

4. **Dividing a Decimal Fraction By a Counting Number :**Divide the given number without considering the decimal point, by the given counting number. Now, in the quotient, put the decimal point to give as many places of decimal as there are in the dividend.
   Suppose we have to find the quotient (0.0204 + 17). Now, 204 ^ 17 = 12. Dividend contains 4 places of decimal. So, 0.0204 + 17 = 0.0012.

5. **Dividing a Decimal Fraction By a Decimal Fraction :**Multiply both the dividend and the divisor by a suitable power of 10 to make divisor a whole number. Now, proceed as above.
   Thus, 0.00066/0.11 = (0.00066*100)/(0.11*100) = (0.066/11) = 0.006V

Comparison of Fractions : Suppose some fractions are to be arranged in ascending or descending order

F. of magnitude. Then, convert each one of the given fractions in the decimal form, and arrange them accordingly.
Suppose, we have to arrange the fractions 3/5, 6/7 and 7/9 in descending order. now, 3/5=0.6,6/7 = 0.857,7/9 = 0.777...
since 0.857>0.777...>0.6, so 6/7>7/9>3/5

G. Recurring Decimal : If in a decimal fraction, a figure or a set of figures is repeated continuously, then such a number is called a recurring decimal. In a recurring decimal, if a single figure is repeated, then it is expressed by putting a dot on it. If a set of figures is repeated, it is expressed by putting a bar on the set
Thus 1/3 = 0.3333….= 0.3; 22 /7 = 3.142857142857.....= 3.142857
Pure Recurring Decimal: A decimal fraction in which all the figures after the decimal point are repeated, is called a pure recurring decimal.

Converting a Pure Recurring Decimal Into Vulgar Fraction : Write the repeated figures only once in the numerator and take as many nines in the denominator as is the number of repeating figures.
thus ,0.5 = 5/9; 0.53 = 53/59 ;0.067 = 67/999;etc...

Mixed Recurring Decimal: A decimal fraction in which some figures do not repeat and some of them are repeated, is called a mixed recurring decimal. e.g., 0.17333 . = 0.173.

Converting a Mixed Recurring Decimal Into Vulgar Fraction : In the numerator, take the difference between the number formed by all the digits after decimal point (taking repeated digits only once) and that formed by the digits which are not repeated, In the denominator, take the number formed by as many nines as there are repeating digits followed by as many zeros as is the number of non-repeating digits.
Thus 0.16 = (16-1) / 90 = 15/19 = 1/6;

0.2273 = (2273 - 22)/9900 = 2251/9900

H. Some Basic Formulae :

(i) $(a + b)(a- b) = (a^2 - b^2)$.
(ii) $(a + b)^2 = (a^2 + b^2 + 2ab)$.
(iii) $(a - b)^2 = (a^2 + b^2 - 2ab)$.
(iv) $(a + b+c)^2 = a^2 + b^2 + c^2+2(ab+bc+ca)$
(v) $(a^3 + b^3) = (a + b) (a^2 - ab + b^2)$
(vi) $(a^3 - b^3) = (a - b) (a^2 + ab + b^2)$.
(vii) $(a^3 + b^3 + c^3 - 3abc) = (a + b + c) (a^2 + b^2 + c^2-ab-bc-ca)$
(viii) When $a + b + c = 0$, then $a^3 + b^3+ c^3 = 3abc$

4. SIMPLIFICATION

A. 'BODMAS' Rule: This rule depicts the correct sequence in which the operations are to be executed, so as to find out the value of a given expression. Here, 'B' stands for 'bracket' ,'O' for 'of' , 'D' for' division' and 'M' for 'multiplication', 'A' for 'addition' and 'S' for 'subtraction'.

Thus, in simplifying an expression, first of all the brackets must be removed, strictly in the order(), {} and [].

After removing the brackets, we must use the following operations strictly in the order:

(1)of (2)division (3) multiplication (4)addition (5)subtraction.

B. Modulus of a real number : Modulus of a real number a is defined as
|a| = a, if a>0 -a, if a<0
Thus, |5|=5 and |-5|=-(-5)=5.

C. Virnaculum (or bar): When an expression contains Virnaculum, before applying the 'BODMAS' rule, we simplify the expression under the Virnaculum.

5. SQUARE ROOTS AND CUBE ROOTS

A. Square Root: If $x^2 = y$, we say that the square root of y is x and we write, $\sqrt{y} = x$. Thus, $\sqrt{4} = 2$, $\sqrt{9} = 3$, $\sqrt{196} = 14$.

B. Cube Root: The cube root of a given number x is the number whose cube is x. We denote the cube root of x by $\sqrt[3]{x}$.
Thus, $\sqrt[3]{8} = \sqrt[3]{2 \times 2 \times 2} = 2$, $\sqrt[3]{343} = \sqrt[3]{7 \times 7 \times 7} = 7$ etc.

Note:
(i) $\sqrt{xy} = \sqrt{x} * \sqrt{y}$
(ii) $\sqrt{(x/y)} = \sqrt{x} / \sqrt{y} = (\sqrt{x} / \sqrt{y}) * (\sqrt{y} / \sqrt{y}) = \sqrt{xy} / y$

6. AVERAGE

An average, or an arithmetic mean, is the sum of `n' different data divided by `n'

Average = Sum of Data / No. of Data
No. of Data = Sum of Data / Average
Sum of Data = Average * No. of Data

Points to remember:

1. **Age of new entrant =** New average + No. of old members x change in average
2. **Age of one who left =** New average - No. of old members x change in average
3. **Age of new person =** Age of the removed person + No. of members x change in average

In all the above three cases, if there is a decrease in the average, the sign of change in average will be negative.

If a certain distance is covered at x km/hr and the same distance is covered by y km/hr, then the average speed during the whole journey is 2xy/(x+y) km/hr.

7. SURDS AND INDICES

A. LAWS OF INDICES:

(i) $a^m \times a^n = a^{m+n}$
(ii) $a^m / a^n = a^{m-n}$
(iii) $(a^m)^n = a^{mn}$
(iv) $(ab)^n = a^n b^n$
(v) $(a/b)^n = (a^n / b^n)$
(vi) $a^0 = 1$

B. SURDS:

Let a be a rational number and n be a positive integer such that $a^{1/n} = {}^n\text{sqrt}(a)$ is irrational. Then ${}^n\text{sqrt}(a)$ is called a surd of order n.

C. LAWS OF SURDS:

(i) ${}^n\sqrt{a} = a^{1/n}$
(ii) ${}^n\sqrt{ab} = {}^n\sqrt{a} * {}^n\sqrt{b}$
(iii) ${}^n\sqrt{a/b} = {}^n\sqrt{a} / {}^n\sqrt{b}$
(iv) $({}^n\sqrt{a})^n = a$
(v) ${}^m\sqrt{({}^n\sqrt{(a)})} = {}^{mn}\sqrt{(a)}$
(vi) $({}^n\sqrt{a})^m = {}^n\sqrt{a^m}$

8. PERCENTAGE

A. Concept of Percentage :A fraction with its denominator as '100' is called a percentage. Percentage means per hundred. So it is a fraction of the form 6/100 , 37/100, 151/100 and these fractions can be expressed as 6%, 37% and 151% respectively. By a certain percent ,we mean that many hundredths.

Thus x percent means x hundredths, written as x%.

To express x% as a fraction : We have , x% = x/100.

Thus, 20% =20/100 =1/5; 48% =48/100 =12/25, etc.

To express a/b as a percent : We have, a/b =((a/b)*100)%.

Thus, ¼ =[(1/4)*100] = 25%; 0.6 =6/10 =3/5 =[(3/5)*100]% =60%.

B. If the price of a commodity increases by R%, then the reduction in consumption so as notto increase the expenditure is
[R / (100+R))*100] %.

If the price of the commodity decreases by R%, then the increase in consumption so as to decrease the expenditure is
[(R/(100-R)*100]%.

C. Results on Population : Let the population of the town be P now and suppose it increases at the rate of R% per annum, then :

1. **Population after n years = P $[1+(R/100)]^n$.**
2. **Population n years ago = P /$[1+(R/100)]^n$.**

D. Results on Depreciation : Let the present value of a machine be P. Suppose it depreciates at the rate R% per annum. Then,

1. **Value of the machine after n years = P$[1-(R/100)]^n$.**
2. **Value of the machine n years ago = P/$[1-(R/100)]^n$.**

E. If A is R% more than B, then B is less than A by
[(R/(100+R))*100]%.

If A is R% less than B , then B is more than A by
[(R/(100-R))*100]%.

9. PROFIT AND LOSS

A. Cost price: the price at which article is purchased. abbreviated as CP.
B. Selling price: the price at which article is sold. abbreviated as SP
C. Profit or gain: if SP is greater than CP, the selling price is said to have profit or gain.
D. Loss: if SP is less than CP, the seller is said to incurred a loss.

E. FORMULA

(i) GAIN=(SP)-(CP).

(ii) LOSS=(CP)-(SP).

(iii) LOSS OR GAIN IS ALWAYS RECKONED ON CP

(iv) GAIN %={GAIN*100}/CP.

(v) LOSS%={LOSS*100}/CP.

(vi) SP={(100+GAIN%) /100}*CP.

(vii) SP={(100-LOSS%)/100}*CP.

(viii) {100/(100+GAIN%)} *SP

(ix) CP=100/(100-LOSS%)}*SP

(x) IF THE ARTICLE IS SOLD AT A GAIN OF SAY 35%, THENSP=135% OF CP

(xi) IF A ARTICLE IS SOLD AT A LOSS OF SAY 35%. THENSP=65% OF CP.

(xii) WHEN A PERSON SELLS TWO ITEMS,ONE AT A GAIN OF X% AND OTHER AT A LOSS OF X%.THEN THE SELLER ALWAYS INCURES A LOSS GIVEN: {LOSS%=(COMON LOSS AND GAIN)^2}/10.=(X/10)^2

IF THE TRADER PROFESSES TO SELL HIS GOODS AT CP BUT USES FALSE WEIGHTS,THENGAIN=[ERROR/(TRUE VALUE)-(ERROR)*100]%

10. RATIO AND PROPORTION

A. RATIO: The ratio of two quantities a and b in the same units, is the fraction a/b and we write it as a:b. In the ratio a:b, we call a as the first term or antecedent and b, the second term or consequent.

Ex. The ratio 5: 9 represents 5/9 with antecedent = 5, consequent = 9.

Rule: The multiplication or division of each term of a ratio by the same non-zero number does not affect the ratio.

Ex. 4: 5 = 8: 10 = 12: 15 etc. Also, 4: 6 = 2: 3.

B. PROPORTION: The equality of two ratios is called proportion.
If a: b = c: d, we write, a: b:: c : d and we say that a, b, c, d are in proportion . Here a and d are called extremes, while b and c are called mean terms.

Product of means = Product of extremes.

Thus, a: b:: c : d <=> (b x c) = (a x d).

C. (i)Fourth Proportional: If a : b = c: d, then d is called the fourth proportional to a, b, c.
(ii)Third Proportional: If a: b = b: c, then c is called the third proportional to a and b.
(iii) Mean Proportional: Mean proportional between a and b is square root of ab

D. (i) COMPARISON OF RATIOS:
We say that (a: b) > (c: d) <=> (a/b)>(c /d).

(ii) COMPOUNDED RATIO:
The compounded ratio of the ratios (a: b), (c: d), (e : f) is (ace: bdf)

E. (i) Duplicate ratio of (a : b) is (a^2 : b^2).
(ii) Sub-duplicate ratio of (a : b) is (√a : √b).
(iii)Triplicate ratio of (a : b) is (^{a3} : ^{b3}).
(iv) Sub-triplicate ratio of (a : b) is (a ⅓ : b ⅓).
(v) If (a/b)=(c/d), then ((a+b)/(a-b))=((c+d)/(c-d)) (Componendo and dividendo)

F. VARIATION:
(i) We say that x is directly proportional to y, if x = ky for some constant k and we write, x ∞ y.
(ii) We say that x is inversely proportional to y, if xy = k for some constant k and we write, x∞(1/y)

11. PARTNERSHIP

A. Partnership: When two or more than two persons run a business jointly, they are called partners and the deal is known as partnership.

B. Ratio of Division of Gains:

i) When investments of all the partners are for the same time, the gain or loss is distributed a among the partners in the ratio of their investments.

Suppose A and B invest Rs. x and Rs. y respectively for a year in a business, then at the end of the year: (A's share of profit) : (B's share of profit) = x : y.

ii) When investments are for different time periods, then equivalent capitals are calculated for a unit of time by taking (capital x number of units of time). Now, gain or loss is divided in the ratio of these capitals.

Suppose A invests Rs. x for p months and B invests Rs. y for q months, then (A's share of profit) : (B's share of profit) = xp : yq.

Working and Sleeping Partners: A partner who manages the business is known .as a working partner and the one who simply invests the money is a sleeping partner.

12. TIME AND WORK

A. If A can do a piece of work in n days, then A's 1 day's work = (1/n).

B. If A’s 1 day's work = (1/n),then A can finish the work in n days.

C. A is thrice as good a workman as B, then:
Ratio of work done by A and B = 3 : 1.
Ratio of times taken by A and B to finish a work = 1 : 3.

D. If the number of men engaged to do a piece of work is changed in the ratio *a:b*, the time required for the work will be changed in the ratio *b:a*

E. If A is X times as good a workman as B, then A will take 1/x of the time that B takes to do a certain work.

F. If M1 persons can do `W1' works in D1 days for T1 hours and M2 persons can do `W2' works in D2 days for T2 hours then M1 D1 T1 W2 = M2 D2 T2W1.

G. If A can finish a work in `x' days and B can finish the same work in `y' days, then time taken by both to finish the work is xy / x+y days

H. If A and B together can do a piece of work in x days and A alone can do it in y days then B alone can do it in xy / y-x days

I. If A, B and C can do a work in x,y, and z days respectively, then all of them working together can finish the work in xyz / xy+yz+zx days

J. If two taps A and B take *a* and *b* hours respectively to fill a tank, then the two taps together fill *1/a + 1/b* part of the tank in an hour and the entire tank is filled in 1/(1/a+1/b) - ab/(a+b)

13. TIME AND DISTANCE

A. Speed, Time and Distance:

Speed = Distance / Time
Time = Distance / Speed
Distance = (Speed x Time)

B. km/hr to m/sec conversion:
x km/hr = x * 5/18 m/sec.

C. m/sec to km/hr conversion:
x m/sec = x * 18/5 km/hr.

D. If the ratio of the speeds of A and B is a : b, then the ratio of the times taken by them to cover the same distance is 1/a : 1/b or b : a.

E. Suppose a man covers a certain distance at x km/hr and an equal distance at y km/hr. Then, the average speed during the whole journey is 2xy / (x+y) km/hr.

14. PROBLEMS ON TRAINS

A. a km/hr= (a* 5/18) m/s.

B. a m / s = (a*18/5) km/hr.

C. Time taken by a train of length 1 meters to pass a pole or a standing man or a signal post is equal to the time taken by the train to cover 1 meters.

D. Time taken by a train of length 1 meters to pass a stationary object of length b meters is the time taken by the train to cover (1 + b) meters.

E. Suppose two trains or two bodies are moving in the same direction at u m / s and v m/s, where u > v, then their relatives speed = (u - v) m / s.

F. Suppose two trains or two bodies are moving in opposite directions at u m / s and v m/s, then their relative speed is = (u + v) m/s.

G. If two trains of length a meters and b meters are moving in opposite directions at u m / s and v m/s, then time taken by the trains to cross each other = (a + b)/(u+v) sec.

H. If two trains of length a meters and b meters are moving in the same direction at u m / s and v m / s, then the time taken by the faster train to cross the slower train = (a+b)/(u-v) sec.

I. If two trains (or bodies) start at the same time from points A and B towards each other and after crossing they take a and b sec in reaching B and A respectively, then (A's speed) : (B's speed) = ($b^{1/2}$: $a^{1/2}$).

15. SIMPLE INTEREST

A. **Principal:** The money borrowed or lent out for a certain period is called the principal or the sum.

B. **Interest:** Extra money paid for using other's money is called interest.

C. **Simple Interest (S.I.) :**If the interest on a sum borrowed for a certain period is reckoned uniformly, then it is called simple interest.

Let Principal = P, Rate = R% per annum (p.a.) and Time = T years. Then,

(i) S.I. =(P*R*T)/100

P=(100*S.I)/(R*T) ;

R=(100*S.I)/(P*T) and

T=(100*S.I)/(P*R)

16. COMPOUND INTEREST

A. **Compound Interest:** Sometimes it so happens that the borrower and the lender agree to fix up a certain unit of time, say yearly or half-yearly or quarterly to settle the previous account.
In such cases, the amount after first unit of time becomes the principal for the second unit, the amount after second unit becomes the principal for the third unit and so on.

B. After a specified period, the difference between the amount and the money borrowed is called the Compound Interest (abbreviated as C.I.) for that period.

Let Principal = P, Rate = R% per annum, Time = n years.

C. When interest is compound Annually:
Amount = $P(1+R/100)^n$

D. When interest is compounded Half-yearly:
Amount = $P[1+(R/2)/100]^{2n}$

E. When interest is compounded Quarterly:
Amount = $P[\ 1+(R/4)/100]^{4n}$

F. When interest is compounded AnnuaI1y but time is in fraction, say 3(2/5) years.
Amount = $P(1+R/100)^3 \times (1+(2R/5)/100)$

G. When Rates are different for different years, say Rl%, R2%, R3% for 1st, 2nd and 3rd year respectively.
Then, Amount = $P(1+R_1/100)(1+R_2/100)(1+R_3/100)$

Present worth of Rs.x due n years hence is given by : **Present Worth = $x/(1+(R/100))^n$**

17. AREA

A. RESULTS ON TRIANGLES:

1. Sum of the angles of a triangle is 180 degrees.

2. Sum of any two sides of a triangle is greater than the third side.

3. Pythagoras theorem:
   In a right angle triangle,$(\text{Hypotenuse})^2 = (\text{base})^2 + (\text{Height})^2$

4. The line joining the midpoint of a side of a triangle to the opposite vertex is called theMEDIAN

5. The point where the three medians of a triangle meet is called CENTROID.Centroid divides each of the medians in the ratio 2:1.

6. In an isosceles triangle, the altitude from the vertex bi-sects the base

7. The median of a triangle divides it into two triangles of the same area.

8. Area of a triangle formed by joining the midpoints of the sides of a given triangle is one-fourth of the area of the given triangle.

B. RESULTS ON QUADRILATERALS:

1. The diagonals of a parallelogram bisects each other .
2. Each diagonal of a parallelogram divides it into two triangles of the same area
3. The diagonals of a rectangle are equal and bisect each other.
4. The diagonals of a square are equal and bisect each other at right angles.
5. The diagonals of a rhombus are unequal and bisect each other at right angles.
6. A parallelogram and a rectangle on the same base and between the same parallels areequal in area.
7. Of all the parallelograms of a given sides , the parallelogram which is a rectangle has thegreatest area.

C. IMPORTANT FORMULAE

1. Area of a rectangle=(length*breadth)
   Therefore length = (area/breadth) and breadth=(area/length)

2. Perimeter of a rectangle = 2*(length + breadth)

3. Area of a square = $(\text{side})^2=1/2(\text{diagonal})^2$

4. Area of four walls of a room = 2*(length + breadth)*(height)

5. Area of the triangle=1/2(base*height)

6. Area of a triangle = $(s*(s-a)(s-b)(s-c))^{1/2}$, where a,b,c are the sides of a triangle and s= ½(a+b+c)

7. Area of the equilateral triangle =((3^1/2)/4)*$(\text{side})^2$

8. Radius of in circle of an equilateral triangle of side a=a/2$(3^{1/2})$

9. Radius of circumcircle of an equilateral triangle of side a=a/($3^{1/2}$)

10. Radius of in circle of a triangle of area del and semi perimeter S=del/S

11. Area of the parellogram =(base *height)

12. Area of the rhombus=1/2(product of the diagonals)

13. Area of the trapezium=1/2(size of parallel sides)*distance between them

14. Area of a circle =pi*r^2,where r is the radius

15. Circumference of a circle = $2\pi R$

16. Length of an arc = $2\pi R\theta/(360)$ where θ is the central angle

17. Area of a sector = (1/2) (arc x R) = pi*R^2*θ/360.

18. Area of a semi-circle = (pi)*R^2.

19. Circumference of a semi-circle = (pi)*R.

18. VOLUME AND SURFACE AREA

A. CUBOID

1. Let length = l , breadth = b and height = h units. Then,
2. Volume = (l x b x h) cubic units.
3. Surface area= 2(lb + bh + lh) sq.units.
4. Diagonal.=$\sqrt{l^2 + b^2 + h^2}$ units

B. CUBE
1. Let each edge of a cube be of length a. Then,
2. Volume = a^3 cubic units.
3. Surface area = $6a^2$ sq. units.
4. Diagonal = $\sqrt{3}$ a units.

C. CYLINDER
1. Let radius of base = r and Height (or length) = h. Then,
2. Volume = ($\pi r^2 h$) cubic units.
3. Curved surface area = ($2\pi rh$). units.
4. Total surface area =$2\pi r(h+r)$ sq. units

D. CONE
1. Let radius of base = r and Height = h. Then,
2. Slant height, l =$\sqrt{h^2+r^2}$
3. Volume = (1/3) $\pi r^2 h$ cubic units.
4. Curved surface area = (πrl) sq. units.
5. Total surface area = ($\pi rl + \pi r^2$) sq. units.

E. SPHERE
1. Let the radius of the sphere be r. Then,
2. Volume = (4/3) πr3 cubic units.
3. Surface area = (4 πr^2) sq. units.

F. HEMISPHERE
1. Let the radius of a hemisphere be r. Then,
2. Volume = (2/3) πr^3 cubic units.
3. Curved surface area = (2 πr^2) sq. units.
4. Total surface area = (3 πr^2) units.

   Remember: 1 litre = 1000 cm3.

19. PERMUTATIONS AND COMBINATIONS

A. FUNDAMENTAL PRINCIPLES OF COUNTING

1. **Multiplication Rule:** If certain thing may be done in 'm' different ways and when it has been done, a second thing can be done in 'n ' different ways then total number of ways of doing both things simultaneously = m × n.

   Eg. if one can go to school by 5 different buses and then come back by 4 different buses, then total number of ways of going to and coming back from school = 5 × 4 = 20.

2. **Addition Rule :**It there are two alternative jobs which can be done in 'm' ways and in 'n'ways respectively then either of two jobs can be done in (m + n) ways.

   Eg.if one wants to go school by bus where there are 5 buses or to by auto where there are4 autos, then total number of ways of going school = 5 + 4 = 9.

   AND => Multiply
   OR => Add

3. **Factorial:** The factorial n, written as n! , represents the product of all integers from 1 to n both inclusive. To make the notation meaningful, when n = o, we define o! = 1.
   Thus, n! = n (n – 1) (n – 2) ….. …3.2.1

   Example : Find 5!, 4! and 6!
   Solution : 5! = 5 × 4 × 3 × 2 × 1 = 120; 4! = 4 × 3 × 2 × 1 = 24; 6! = 6 × 5 × 4 × 3 × 2 × 1 = 720.

B. Permutation: The ways of arranging or selecting smaller or equal number of persons or objects from a group of persons or collection of objects with due regard being paid to the order of arrangement or selection, are called permutations.

Example :A group of persons want themselves to be photographed. They approach the photographer and request him to take as many different photographs as possible with persons standing in different positions amongst themselves. The photographer wants to calculate how many films does he need to exhaust all possibilities? How can he calculate the number?

In the situations such as above, we can use permutations to find out the exact number of films.

Solution : Let us explain, how the idea of permutation will help the photographer. Suppose the group consists of Mr. Suresh, Mr. Ramesh and Mr. Mahesh. Then how many films does the photographer need? He has to arrange three persons amongst three places with due regard to order. Then the various possibilities are (Suresh, Mahesh, Ramesh), (Suresh, Ramesh, Mahesh), (Ramesh, Suresh, Mahesh), (Ramesh, Mahesh, Suresh), (Mahesh, Ramesh, Suresh) and (Mahesh, Suresh, Ramesh). Thus there are six possibilities. Therefore he needs six films. Each one of these possibilities is called a permutation of three persons taken at a time.

The different arrangements of a given number of things by taking some or all at a time, are called permutations.

Ex. 1.All permutations (or arrangements) made with the letters a, b, c by taking two at a time are: (ab, ba, ac, bc, cb).

Ex. 2.All permutations made with the letters a,b,c, taking all at a time are: (abc, acb, bca, cab, cba).

Number of Permutations: Number of all permutations of n things, taken r at a time, given by:

$$^nP_r = n(n-1)(n-2).....(n-r+1) = n!/(n-r)!$$

Examples: (i) $^6p_2 = (6x5) = 30$. (ii) $^7p_3 = (7x6x5) = 210$.

Cor. Number of all permutations of n things, taken all at a time = n!

An Important Result: If there are n objects of which p_1 are alike of one kind; p_2 are alike of another kind; p_3 are alike of third kind and so on and p_r are alike of rth kind, such that $(p_1+p_2+... .p_r) = n$.

Then, number of permutations of these n objects is:

$$n! \ / \ (p_1!).p_2!)......(pr!)$$

C. Combinations: Each of the different groups or selections which can be formed by taking some or all of a number of objects, is called a combination.

We have studied about permutations in the earlier section. There we have said that while arranging, we should pay due regard to order. There are situations in which order is not important. For example, consider selection of 5 clerks from 20 applicants. We will not be concerned about the order in which they are selected. In this situation, how to find the number of ways of selection? The idea of combination applies here.

Definition : The number of ways in which smaller or equal number of things are arranged or selected from a collection of things where the order of selection or arrangement is not important, are called combinations.

Ex. 1. Suppose we want to select two out of three boys A, B, C. Then, possible selections are AB, BC and CA.
Note that AB and BA represent the same selection.

Ex. 2. All the combinations formed by a, b, c, taking two at a time are ab, bc, ca.

Ex. 3. The only combination that can be formed of three letters a, b, c taken all at a time is abc.
Ex. 4. Various groups of 2 out of four presons A, B, C, D are: AB, AC, AD, BC, BD, CD.

Ex. 5. Note that ab and ba are two different permutations but they represent the same combination.

Number of Combinations: The number of all combination of n things, taken r at a time is:

$\mathbf{{}^nC_r = n! / (r!)(n-r)! = n(n-1)(n-2).....to\ r\ factors / r!}$

Note that: ${}^nC_r = 1$ and ${}^nC_0 = 1$.

An Important Result: $\mathbf{{}^nC_r = {}^nC_{(n-r)}}$.

Example:
(i) ${}^{11}C_4 = (11x10x9x8)/(4x3x2x1) = 330$.
(ii) ${}^{16}C_{13} = {}^{16}C_{(16-13)} = 16x15x14/3! = 16x15x14/3x2x1 = 560$.

20. PROBABILITY

A. **Experiment :** An operation which can produce some well-defined outcome is called an experiment

B. **Random experiment:** An experiment in which all possible outcome are known and the exact out put cannot be predicted in advance is called an random experiment

Examples of performing random experiment:

(i) rolling an unbiased dice
(ii) tossing a fair coin
(iii) drawing a card from a pack of well shuffled card
(iv) picking up a ball of certain color from a bag containing ball of different colors

Details:

(i) When we throw a coin. Then either a head(h) or a tail (t) appears.
(ii) A dice is a solid cube, having 6 faces ,marked 1,2,3,4,5,6 respectively when we throw a die , the outcome is the number that appear on its top face .
(iii) A pack of cards has 52 cards it has 13 cards of each suit ,namely Spades, Clubs , Hearts and Diamonds. Cards of spades and clubs are black cards. Cards of hearts and diamonds are red cards There are 4 honors of each suit. These are Aces, King , Queen and Jack. These are called face cards.

C. **Sample space :** When we perform an experiment ,then the set S of all possible outcome is called the sample space. Denoted by 's'

Example of sample space:

(i) in tossing a coin ,s={h,t}
(ii) if two coin are tossed ,then s={hh,tt,ht,th}.
(iii) in rolling a die we have, s={1,2,3,4,5,6}.

D. Event: Any subset of a sample space.

E. Probability of occurrence of an event.

let S be the sample space and E be the event . then, $E \subseteq S$.
$P(E)=n(E)/n(S)$.

F. Results on Probability:

(i) $P(S) = 1$
(ii) $0<P(E)<1$
(iii) $P(\varphi)=0$
(iv) For any event a and b, we have: $P(a \cup b)=P(a)+P(b)-P(a \cup b)$
(v) If A denotes (not-a),then $P(A)=1-P(A)$.